Linear dispersion relation for a fluid of light in an atomic vapor

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We report here the first measurement of the sound-like dispersion of density waves propagating on top of a correlated photons fluid generated by the 3rd order Kerr non-linearity in a hot Rb^{85} vapor.

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In 2000, R. Chiao published a paper asking if a photon fluid can be considered as a superfluid ? [1] Indeed, there are strong formal analogies between the Non-Linear Schrodinger Equation (NLSE) describing a beam of light propagating in a Kerr non-linear medium and the Gross-Pitaevskii Equation (GPE) describing the mean field behaviour of a quantum Bose gas. Superfluidity is an universal phenomena which has been observed in a wide variety of systems from liquid Helium to atomic condensates. Superfluidity of light has also been reported using exciton-polaritons in a microcavity. Here we report the first observation of a linear dispersion relation for a fluid of light propagating in a hot atomic vapor addressing the question ask by R. Chiao almost two decades ago. We believe that this demonstration can open the way for a new field of research on quantum fluid of light in the propagating geometry [2].

I. PROPAGATION OF AN INTENSE LASER FIELD IN AN OPTICAL KERR NONLINEAR MEDIUM

The wave equation describing the evolution in a Kerr defocusing nonlinear medium of the slow-varying envelope $E\,(\mathbf{r}_\perp,\, z)$ of a monochromatic laser field of frequency ω_0, can be written as follow (in the paraxial approximation) :

$$\frac{\partial E}{\partial z} = \frac{i}{2\,k_0}\,\nabla_\perp^2\,E - i\,\frac{k_0\,n_2}{n_0}\,|E|^2\,E\,, \tag{1}$$

where $k_0 = n_0\,c/\omega_0$, n_0 the linear refractive index and $n_2 = 3\,\chi^{(3)}/8\,n_0$ the 3rd order Kerr susceptibility. $\nabla_\perp$ stands for the gradient in the transverse plane. This equation is mathematically identical to the GPE in 2D, describing the dynamic of the mean field wave function in a Bose–Einstein condensate (BEC). The propagation distance z inside the cell can then be seen as an effective time of evolution for the correlated photons gas, usually called fluid of light.

This analogy between NLSE and GPE naturally leads one to ask if superfluidity of light can be observed in that kind of system, as suggested in [1]. In this paper we study small amplitude excitations on top of the fluid of light to measure the dispersion relation of the fluid of light. In a BEC, the dispersion of that kind of small amplitude density waves is given by the Bogoliubov formula :

$$\Omega_B\,(k_\perp) = \sqrt{\frac{g\,\rho_0}{m}\,(\hbar\,k_\perp)^2 + \left(\frac{\hbar\,k_\perp}{2\,m}\right)^2}\,. \tag{2}$$

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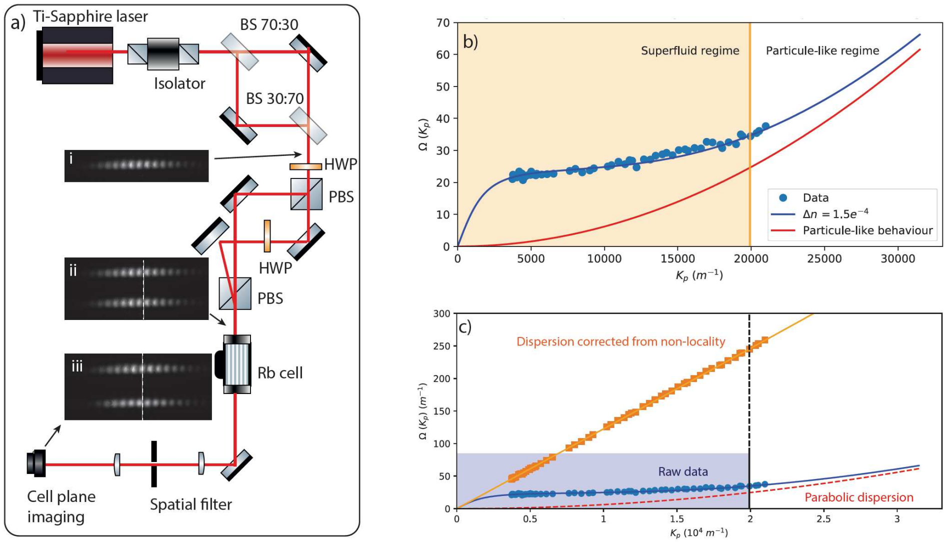

Figure 1. (a) : A Mach-Zehnder Interferometer splits a collimated elliptical gaussian beam (elongated along the X-axis) into a pump and a probe beam. Vertical interference fringes (along Y-axis) are created and the fringes spacing can be tuned changing the angle between pump and probe. The intensity modulated gaussian beam obtained is split again into a reference and a hight power part, disposed one above the other (upper fringes aligned with lower ones). Both beams are sent through the hot atomic vapor ($T = 410$ K). The output plane of the cell is directly imaged on the camera. (b) and (c) : Dispersion $\Omega_B(k_\perp)$. As predicted, the behaviour is quadratic (particle-like behaviour) for large $k_\perp$ values, however the linear regime expected at low $k_\perp$ is strongly modified. The blue curve corresponds to the best fit obtained adding the nonlocal reponse function.

$g = \hbar k_\perp c n_2 / n_0^2$ stands for the interaction coupling constant , $\rho_0 = |E|^2$ for the density of the Bose gas and $m = \hbar k_\perp n_0 / c$ for the mass of the bosons involved. For low $k_\perp$ values, the relation becomes linear and excitations follow a sound-like behaviour characterised by the sound velocity $c_S = \sqrt{g\,\rho_0/n}$.

II. EXPERIMENTAL MEASUREMENT OF THE SOUND-WAVES DISPERSION

The goal of our experiment is mainly to measure the dispersion of sound-waves excited on top of a fluid of light. In order to do so, we send two intensity modulated highly elliptical gaussian beams, a powerful one (HP) above a reference (LP), at a way lower intensity, as described in figure (1a). Both beams are red-detuned with respect to the Rb^{85} D_2 line ($\delta = -4$ GHz).

The intensity modulation on both beams can be seen as a density wave moving on top of a fluid of light. The phase velocity v_{ph} of this wave depends in the nonlinearity and increases with the field intensity. In the output plane, the wave moving on the HP beam travels a bigger distance than the one moving on LP one, resulting in a shift ΔS between the fringes of the upper and lower interference paterns. This shift can be expressed as follows :

$$\Delta S = \left(v_{ph}^{HP} - v_{ph}^{LP}\right) L = \left(\Omega_B^{HP}(k_\perp) - \Omega_B^{LP}(k_\perp)\right) \frac{L}{k_\perp} = \frac{k_\perp}{2\,k_0} \left(\sqrt{1 + \frac{\Delta n}{n_0} \left(\frac{2\,k_0}{k_\perp}\right)} - 1 \right) L\,, \tag{3}$$

where L stands for the length of the cell, $k_\perp$ for the density wave k-vector in the transverse plane and $\Delta n = n_2\,|E|^2$ for the nonlinear refractive index. By measuring the shift ΔS for different angles (i.e. different wavelengths), we reconstruct the dispersion $\Omega_B(k_\perp)$. This measurement has been reported in figure (1b). The shape of the dispersion obtained is different from the one predicted by Chiao in [1]. However, the effective photon-photon interaction is non-local in our system. Including this effect will enable us to reconcile our results with the prediction of [1].

III. NONLOCAL PHOTON-PHOTON INTERACTIONS

The same kind of experiment has been performed in a thermo-optical defocusing nonlinear media [2]. Highly nonlocal behaviour was observed with these systems, because of heat conduction inside the medium. The nonlinear refractive index $\Delta n(x,\,y)$ does not only depend on the intensity at $(x,\,y)$ but also on surrounding field intensities. Δn has then to be convolved with a given nonlocal response function R. The effective photon-photon interaction in our experiment is mediated by atoms which experience a ballistic transport into the cell and hence, apperas to be intrinsically non-local as well. Including in the shift formula the response function proposed in [3] for hot alkaline vapors enables us to fit correctly the dispersion figure (1b) and (1c) (blue curve).